

Univerzitet u Nišu, Tehnološki Fakultet, Leskovac

PRIJEMNI ISPIT IZ MATEMATIKE, 30.6.2020.

Zadaci

1. Skratiti razlomak i zapisati uslove pod kojima dobijene jednakosti važe:

$$\frac{a^2 + ab + a + b}{a^2 + 2ab + b^2}.$$

2. Izvršiti naznačene operacije:

$$\left(\frac{a}{b} + \frac{b}{a}\right) : \left(\frac{a}{b} + \frac{b}{a} + 2\right) + \frac{2}{ab} : \left(\frac{1}{a} + \frac{1}{b}\right)^2.$$

3. Rešiti jednačinu:

$$\log(x-1) + 2\log\sqrt{x+2} = 1.$$

4. Izračunati vrednost izraza:

$$\left[\left(-\frac{27}{8}\right)^{-\frac{1}{3}} \left(\frac{81}{16}\right)^{-\frac{3}{4}} \left(-\frac{3}{2}\right)^{-2} \right]^{-\frac{3}{5}}.$$

5. Rešiti jednačinu:

$$4^{\sqrt{x-2}} + 16 = 10 \cdot 2^{\sqrt{x-2}}.$$

Rešenja

$$1. \quad \frac{a^2 + ab + a + b}{a^2 + 2ab + b^2} = \frac{a(a+b) + (a+b)}{(a+b)^2} = \frac{(a+b)(a+1)}{(a+b)^2} = \frac{a+1}{a+b}, \quad a+b \neq 0.$$

$$2. \quad \begin{aligned} \left(\frac{a}{b} + \frac{b}{a}\right) : \left(\frac{a}{b} + \frac{b}{a} + 2\right) + \frac{2}{ab} : \left(\frac{1}{a} + \frac{1}{b}\right)^2 &= \frac{a^2 + b^2}{ab} : \frac{a^2 + b^2 + 2ab}{ab} + \frac{2}{ab} : \left(\frac{b+a}{ab}\right)^2 \\ &= \frac{a^2 + b^2}{ab} \cdot \frac{ab}{(a+b)^2} + \frac{2}{ab} \cdot \frac{a^2 b^2}{(a+b)^2} = \frac{a^2 + b^2}{(a+b)^2} + \frac{2ab}{(a+b)^2} \\ &= \frac{a^2 + 2ab + b^2}{(a+b)^2} = \frac{(a+b)^2}{(a+b)^2} = 1, \quad a \neq 0, \quad b \neq 0, \quad a+b \neq 0. \end{aligned}$$

$$3. \quad \begin{aligned} \log(x-1) + 2 \log \sqrt{x+2} &= 1, \\ \log(x-1) + \log(x+2) &= 1, \quad x > 1, \quad x > -2, \\ \log((x-1)(x+2)) &= 1, \\ (x-1)(x+2) &= 10, \quad x^2 + x - 12 = 0, \quad x_{1/2} = \frac{-1 \pm \sqrt{1+48}}{2} = \frac{-1 \pm 7}{2}, \quad x_1 = 3. \end{aligned}$$

$$4. \quad \begin{aligned} &\left[\left(-\frac{27}{8}\right)^{-1/3} \left(\frac{81}{16}\right)^{-3/4} \left(-\frac{3}{2}\right)^{-2} \right]^{-3/5} \\ &= \left[\left(-\frac{8}{27}\right)^{1/3} \left(\frac{16}{81}\right)^{3/4} \left(-\frac{2}{3}\right)^2 \right]^{-3/5} \\ &= \left[\left(-\frac{2}{3}\right) \left(\frac{2}{3}\right)^3 \left(-\frac{2}{3}\right)^2 \right]^{-3/5} = \left(-\frac{2^6}{3^6}\right)^{-3/5} \\ &= \left(-\frac{3^6}{2^6}\right)^{3/5} = - \left(\left(\frac{3}{2}\right)^{18} \right)^{1/5} = - \left(\frac{3}{2}\right)^3 \left(\frac{3}{2}\right)^{3/5}. \end{aligned}$$

5. $4\sqrt{x-2} + 16 = 10 \cdot 2\sqrt{x-2}$,
 $2^{2\sqrt{x-2}} + 16 = 10 \cdot 2\sqrt{x-2}$, *smena* : $2^{\sqrt{x-2}} = t$, $x \geq 2$, $t > 0$;
 $t^2 - 10t + 16 = 0$, $t_{1/2} = \frac{10 \pm \sqrt{100 - 64}}{2} = \frac{10 \pm 6}{2}$, $t_1 = 8$, $t_2 = 2$,
 $2^{\sqrt{x-2}} = 8$, *sledi* $2^{\sqrt{x-2}} = 2^3$, $\sqrt{x-2} = 3$, $x - 2 = 9$, $x = 11$;
 $2^{\sqrt{x-2}} = 2$, *sledi* $\sqrt{x-2} = 1$, $x - 2 = 1$, $x = 3$.